Student Workbook

Unit 12 Review — Limits, Continuity & Derivatives

Name: ________________________________ Date: _________________ Class: _________

UNIT SUMMARY

Consolidate your mastery of estimating limits, applying limit laws, testing continuity, and computing derivatives from the definition — the complete foundation for AP Calculus.

Unit 12 at a Glance: Calculus Preview

$$f'(x) = \lim_{h\to 0} [f(x+h) - f(x)] / h$$

12.1	Estimating Limits — table / graph approach; one-sided limits lim $x\to a^-$ and $x\to a^+$; limit exists iff both sides agree.
12.2	Limit Laws — sum, product, quotient, power rules; direct substitution; factor or rationalize 0/0 indeterminate forms.
12.3	Continuity — three conditions: f(a) defined, lim exists, lim = f(a). Types: removable, jump, infinite. IVT guarantees a root on [a,b].
12.4	Derivatives — $f'(x) = \lim_{h\to 0} [f(x+h)-f(x)]/h$. Tangent line: $y - f(a) = f'(a)(x - a)$.

Key Vocabulary Review

Limit	The value a function approaches as the input approaches a given value, written $\lim(x\to a) f(x) = L$. Ex: $\lim(x\to 2)(x^2-4)/(x-2) = 4$.
One-Sided Limit	A limit approached from only one direction: left ($x\to a^-$) or right ($x\to a^+$). Ex: $\lim(x\to 0^+)\ 1/x = +\infty$.
Continuity	f is continuous at x=a if: (1) f(a) defined, (2) $\lim(x\to a) f(x)$ exists, (3) $\lim(x\to a) f(x) = f(a)$.
Derivative	The instantaneous rate of change of f at x: $f'(x) = \lim(h\to 0) [f(x+h)-f(x)]/h$. Ex: $f(x)=x^2 \to f'(x)=2x$.
Tangent Line	The line touching a curve at exactly one point with slope equal to the derivative there.
IVT	If f is continuous on [a,b] and N is between f(a) and f(b), there exists c in (a,b) with f(c)=N.

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Worked Examples

EXAMPLE 1

Estimate lim(x→2) $(x^2-4)/(x-2)$ using a numerical table

- Direct substitution gives 0/0, so build a table approaching x=2 from both sides
- x=1.9 → 3.9; x=1.99 → 3.99; x=2.01 → 4.01; x=2.1 → 4.1
- Both sides approach 4

Answer: lim(x→2) $(x^2-4)/(x-2) = 4$

EXAMPLE 2

Evaluate lim(x→3) $(x^2-9)/(x-3)$ using limit laws and factoring

- Direct substitution: 0/0 — indeterminate form
- Factor: $x^2-9 = (x-3)(x+3)$. Cancel the common factor $(x-3)$, valid since $x\neq 3$
- Simplified: x+3. Substitute: 3+3

Answer: lim(x→3) $(x^2-9)/(x-3) = 6$

EXAMPLE 3

Determine whether f(x) = {2x if x≤1; x^2+1 if x>1} is continuous at x=1

- f(1) defined: f(1)=2(1)=2 ✓
- Left: lim($x\to 1^-$) 2x=2. Right: lim($x\to 1^+$) $(x^2+1)=2$. Both sides equal 2 ✓
- lim = f(1): 2=2 ✓ — all three conditions satisfied

Answer: f is continuous at x = 1

EXAMPLE 4

Use the limit definition to find f′(x) for f(x) = $x^2 + 3x$

- f′(x) = lim(h→0) [f(x+h)−f(x)]/h. f(x+h) = $x^2+2xh+h^2+3x+3h$
- f(x+h)−f(x) = $2xh+h^2+3h$. Divide by h: 2x+h+3
- Take the limit as h→0: 2x+0+3

Answer: f′(x) = 2x + 3

EXAMPLE 5

Find the tangent line to f(x) = x^2 at x = 2, then verify using the derivative

- f(2)=4, so the point of tangency is (2,4)
- f′(x) = lim(h→0) $[(x+h)^2-x^2]/h = 2x$. f′(2) = 4
- Point-slope: y−4 = 4(x−2) → y = 4x−8+4

Answer: Tangent line: y = 4x − 4

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Estimate lim(x→0) (sin 2x)/x using a table of values.

Hint: Try x = ±0.1, ±0.01, ±0.001. Remember sin 2x ≈ 2x for small x, so (sin 2x)/x ≈ 2.

2 Evaluate lim(x→−3) $(x^2+5x+6)/(x+3)$ by factoring.

Hint: Factor the numerator: $x^2+5x+6 = (x+2)(x+3)$. Cancel (x+3), then substitute x=−3.

3 Find the value of k so that f(x) = {kx+2 if x≤1; 3x−1 if x>1} is continuous at x=1.

Hint: Set the left-hand limit equal to the right-hand limit at x=1: k(1)+2 = 3(1)−1. Solve for k.

4 Use the limit definition to find f′(2) for $f(x) = 3x^2$.

Hint: f′(2) = lim(h→0) $[3(2+h)^2−3(4)]/h$. Expand $(2+h)^2$, simplify, then cancel h before taking the limit.

5 Write the equation of the tangent line to $f(x) = x^3$ at x = 1.

Hint: Find f(1)=1. Use the limit definition to show $f′(x)=3x^2$, so f′(1)=3. Then use y−1=3(x−1).

Interactive Practice

Circle the letter of the correct answer for each question.

1 **Which method is used to evaluate lim(x→5) $(x^2−25)/(x−5)$?** Multiple Choice

- **A** Direct substitution (gives 0/0, so the limit is 0)
- **B** Factor $x^2−25=(x−5)(x+5)$, cancel (x−5), then substitute
- **C** Build a table — algebraic methods never work here
- **D** The limit does not exist because of the 0/0 form

2 **A function f has f(2)=5, lim($x→2^-$) f(x)=5, and lim($x→2^+$) f(x)=3. What can you conclude?** Multiple Choice

- **A** f is continuous at x=2 because f(2) is defined.
- **B** f has a removable discontinuity at x=2.
- **C** f has a jump discontinuity at x=2.
- **D** f has an infinite discontinuity at x=2.

3 **Using the limit definition, what is $f'(x)$ for $f(x) = 4x$?** Multiple Choice

- **A** 0
- **B** 4x
- **C** 4
- **D** 4h

4 **The tangent line to $f(x) = x^2 + 1$ at $x = 3$ has slope:** Multiple Choice

- **A** 3
- **B** 6
- **C** 10
- **D** 9

5 **The Intermediate Value Theorem guarantees a root of f on (a,b) when:** Multiple Choice

- **A** f(a) and f(b) are both positive.
- **B** f is continuous on [a,b] and $f(a)\cdot f(b) < 0$.
- **C** f(a)=0 or f(b)=0.
- **D** f is differentiable on (a,b).

Common Mistakes to Avoid

✗ Substituting directly into 0/0 and writing the limit is 0.	✓ A 0/0 result means the form is indeterminate — factor, rationalize, or use a table.
✗ Checking only one condition (e.g., just that f(a) is defined) to claim continuity.	✓ All three conditions must hold: f(a) defined, limit exists, and limit equals f(a).
✗ Forgetting to divide by h before taking the limit in the derivative definition.	✓ Always simplify $[f(x+h)-f(x)]/h$ so h cancels before substituting h=0.
✗ Using the wrong point in the tangent line formula (e.g., using (0,0) instead of (a,f(a))).	✓ The tangent line passes through (a,f(a)) with slope $f'(a)$: $y-f(a) = f'(a)(x-a)$.

Math Tips

★ When estimating limits numerically, use at least three values on each side and look for convergence.

★ For 0/0 forms, always try factoring first — it works for most polynomial expressions.

★ Check all three continuity conditions in order; if any fails, identify the type of discontinuity.

★ The derivative definition always starts with $f(x+h) - f(x)$ in the numerator — expand carefully.

★ A tangent line has exactly one slope ($f'(a)$) and passes through exactly one point (a, f(a)).